On the Piecewise Linear Unsplittable Multicommodity Flow Problem

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DRCN 2016, Paris, March 15th, 2016

OUTLINE

PROBLEM DEFINITION

- Multicommodity Flow Problem:
  - Graph $G = (V, A)$;
  - Flow capacity on the arcs, C_a;
  - Cost $g_a(l_a)$ on the flow of each arc, l_a;
  - Set of commodities K, each $k \in K$ with a given origin o_k, destination d_k and demand ρ_k;
  - How to route the commodities in G, such that the total cost, $\sum_{a \in A} g_a(l_a)$ is minimized?

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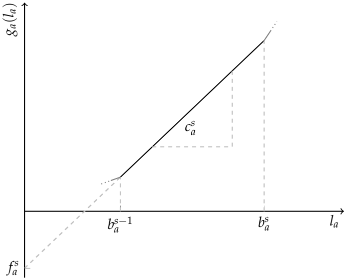
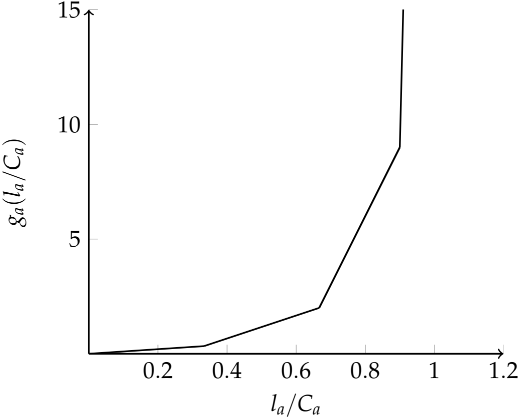
- Single path routing for each commodity: the flows are unsplittable.
- Flow cost functions g_a are piecewise linear:
  - Convex: The Convex Piecewise Linear Unsplittable Multicommodity Flow (PUMF) Problem.

CONVEX PIECEWISE LINEAR FUNCTIONS



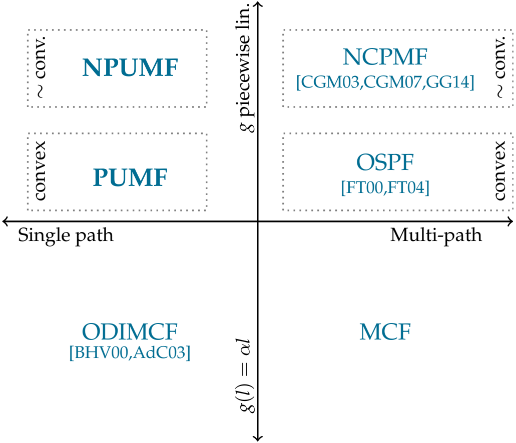
- $S_a = \{1, 2, ..., |S_a|\}$;
- $c_a^1 \geq 0$ and $c_a^s > c_a^{s-1}$;
- $f_a^s \leq 0$ and $f_a^s < f_a^{s-1}$;
- g_a is continuous: $b_a^s c_a^s + f_a^s = b^s c_a^{s+1} + f_a^{s+1}$;
- $g_a(0) = 0$, and consequently, $f_a^1 = 0$.

EXAMPLE OF CONVEX PIECEWISE LINEAR FUNCTION



in Fortz, Thorup (2000, 2004)

STATE OF THE ART (MIP/LP)



CGM : Croxton, Gendron and Magnanti; GG : Gendron and Gouveia; FT : Fortz and Thorup; BHV : Barnhart, Hane and Vance; AdC : Alvelos and de Carvalho.

COMPLEXITY OF THE PUMF PROBLEM

Theorem 1: The PUMF problem is polynomially solvable for $|K| = 1$.

Proof: Reduction to shortest path problem.

Theorem 2: The PUMF problem is $\mathcal{NP}$-hard for $|K| > 1$.

Proof: Reduction to bin packing problem.

BASIC MODEL 2 (BM2)

- $x_a^k = 1$ if arc $a \in A$ is on the unique path chosen to route commodity $k \in K$, and $x_a^k = 0$ otherwise;
- l_a = total load travelling through arc $a \in A$;
- g_a = routing cost of arc $a \in A$.

$$\min \quad \sum_{a \in A} g_a \tag{1a}$$

$$\text{s.t.} \quad \sum_{a \in \delta^+(i)} x_a^k - \sum_{a \in \delta^-(i)} x_a^k = \lambda_i^k, \qquad i \in V, k \in K, \tag{1b}$$

$$l_a = \sum_{k \in K} \rho^k x_a^k, \qquad a \in A, \tag{1c}$$

$$g_a \geq f_a^s + c_a^s l_a, \qquad a \in A, s \in S_a, \tag{1d}$$

$$x_a^k \in \{0, 1\}, \qquad a \in A, k \in K, \tag{1e}$$

$$l_a \geq 0, \qquad a \in A, s \in S_a, \tag{1f}$$

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\min \quad & \sum_{a \in A, s \in S_a} (f_a^s y_a^s + c_a^s l_a^s) && && \text{(2a)}\\
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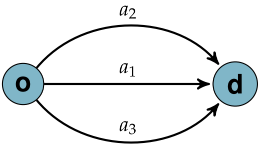
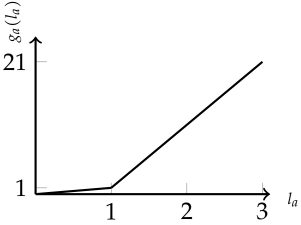


$\rho^{o \to d} = 3$

$C_{a_1} = C_{a_2} = C_{a_3} = 3$



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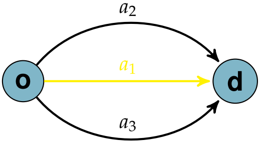

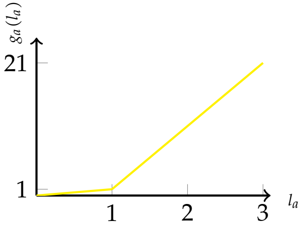

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0-1 solution:
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$g^* = 21$

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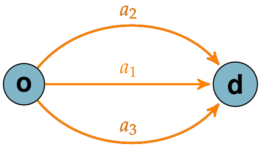

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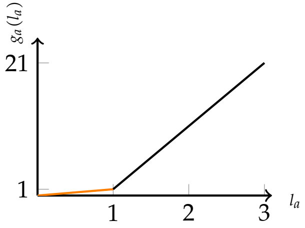

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LP solution:
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$g^* = 3$

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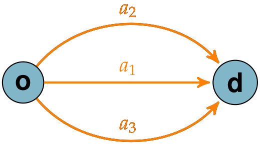

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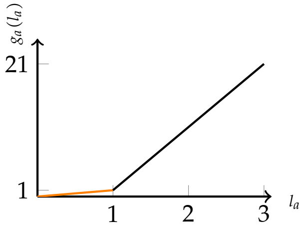

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$g^* = 3$

$gap_{LP} \simeq 86\%$

Disaggregated Model (DM)

- y_a^s;
- $x_a^{ks} = 1$ if arc $a \in A$, which is on segment $s \in S_a$, is on the unique path chosen to route commodity $k \in K$, and $x_a^{ks} = 0$ otherwise.

$$\min \sum_{a \in A, s \in S_a} \left(f_a^s y_a^s + c_a^s \sum_{k \in K} \rho^k x_a^{ks} \right) \quad (3a)$$

$$\text{s.t.} \sum_{a \in \delta^+(i), s \in S_a} x_a^{ks} - \sum_{a \in \delta^-(i), s \in S_a} x_a^{ks} = \lambda_i^k, \quad i \in V, k \in K, \quad (3b)$$

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$$b_a^{s-1} y_a^s \leq \sum_{k \in K} \rho^k x_a^{ks} \leq b_a^s y_a^s, \quad a \in A, s \in S_a, \quad (3d)$$

$$x_a^{ks} \in \{0,1\}, \quad a \in A, k \in K, s \in S_a, \quad (3e)$$

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STRONG MODEL (SM)

Valid inequalities:

$$x_a^{ks} = 0, \qquad a \in A, k \in K, s \in S_a : b_a^s < \rho^k \quad (4)$$

$$b_a^{s-1} y_a^s \leq \sum_{k \in K} \min(\rho^k, b_a^{s-1}) x_a^{ks}, \qquad a \in A, s \in S_a. \quad (5)$$

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DM + (4) + (5) = Strong Model (SM)

Theorem 3: The LP feasible set of the SM provides a complete description of the associated polyhedron for $|K| = 1$.

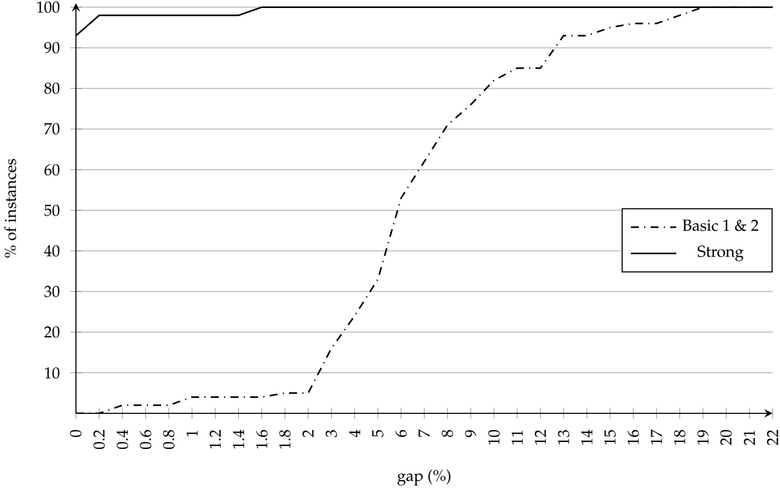
TEST SET T_1

Randomly generated instances:

- 5 instances per class ($|V|$, $|A|$, $|K|$);
- $C_a = \{50, 75, 100\}, a \in A$;
- $\rho^k = \alpha O_{o(k)} D_{d(k)} R_{(o(k),d(k))} e^{\frac{-L_2(o(k),d(k))}{2\Delta}}, k \in K$;
- routing costs given by function in Fortz, Thorup (2000, 2004).

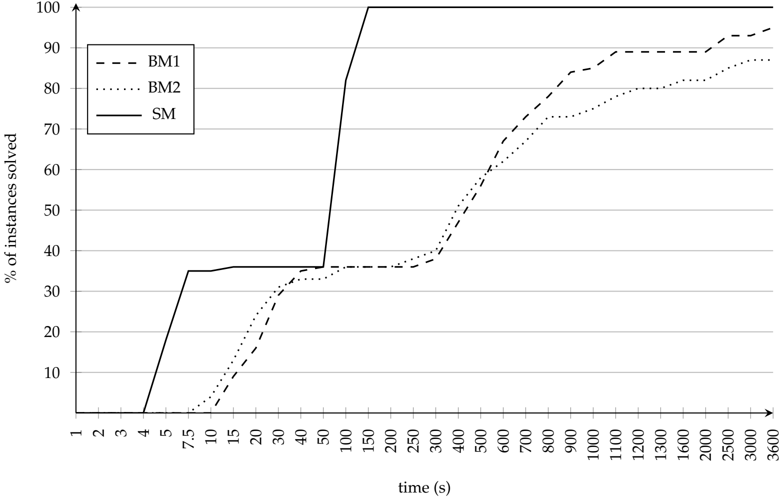
Cl	1	2	3	4	5	6	7	8	9	10	11
$\lvert V\rvert$	40	40	40	60	60	60	80	80	80	80	80
$\lvert A\rvert$	936	1092	1092	2478	2832	2832	316	1896	1896	3160	1896
$\lvert K\rvert$	70	70	100	200	150	200	250	200	250	200	350

LP GAP

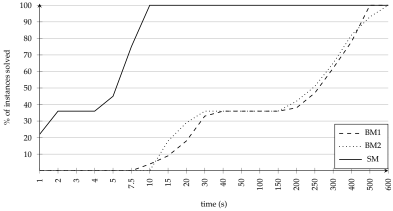


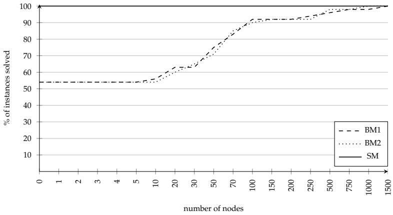
100
90
80
70
60
50
40
30
20
10
% of instances
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
gap (%)
Basic 1 & 2
Strong

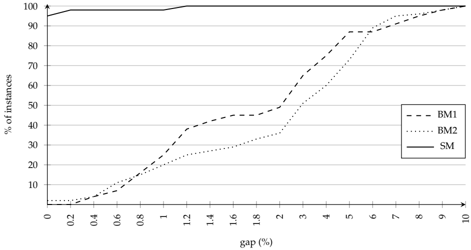
MIP TIME


100
90
80
70
60
50
40
30
20
10
% of instances solved
BM1
BM2
SM
1 2 3 4 5 7.5 10 15 20 30 40 50 100 150 200 250 300 400 500 600 700 800 900 1000 1100 1200 1300 1600 2000 2500 3000 3600
time (s)

LP TIME, B&B NODES, ROOT NODE GAP







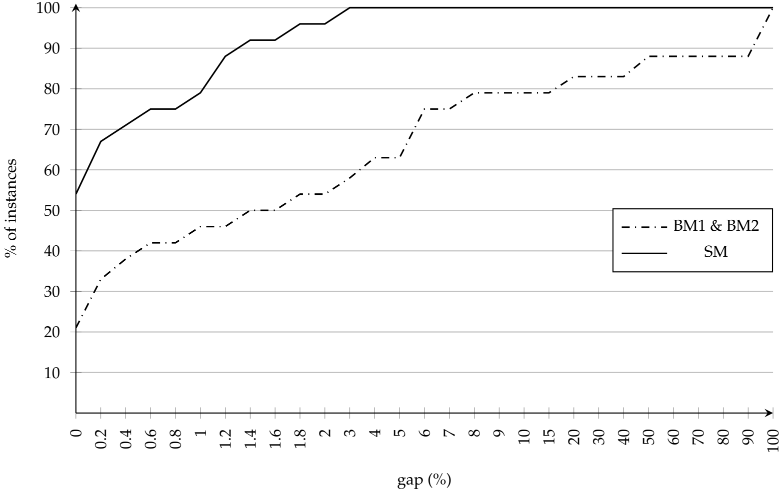
TEST SET T_2

Adapted 4 instances from each 1 of `SNDlib`:

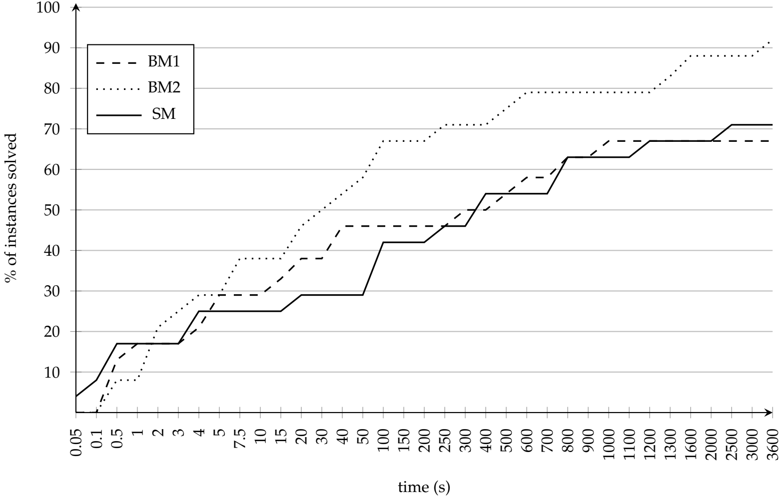
- C_a = `pre-installed capacity` or `first capacity module`, $a \in A$.
- increased/decreased C_a proportionally such that:
  - average arc utilization is (33+5)%;
  - average arc utilization is (66+5)%;
  - average arc utilization is (100+5)%.
- routing costs given by function in Fortz, Thorup (2000, 2004).

Instance ID	`atlanta`	`france`	`newyork`	`pdh`	`sun`	`ta1`
$\lvert V \rvert$	15	25	16	11	27	24
$\lvert A \rvert$	44	90	98	68	102	110
$\lvert K \rvert$	210	300	240	24	67	396

LP GAP

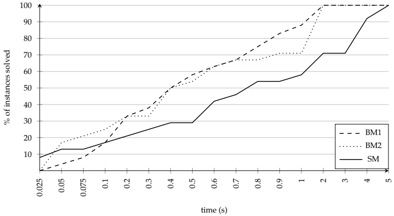

100
90
80
70
60
50
40
30
20
10
% of instances
BM1 & BM2
SM
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
3
4
5
6
7
8
9
10
15
20
30
40
50
60
70
80
90
100
gap (%)

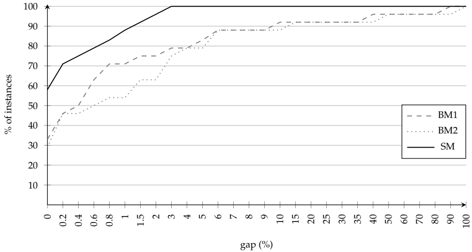
MIP TIME



100
90
80
70
60
50
40
30
20
10
BM1
BM2
SM
% of instances solved
0.05
0.1
0.5
1
2
3
4
5
7.5
10
15
20
30
40
50
100
150
200
250
300
400
500
600
700
800
900
1000
1100
1200
1300
1600
2000
2500
3000
3600
time (s)

LP TIME, B&B NODES, ROOT NODE GAP





SUMMARY AND CONCLUSIONS

- We tackled the novel multicommodity flow problem with unsplittable flows and convex piecewise linear costs;
- We showed that the problem is $\mathcal{NP}$-hard for $|K| > 1$, but polynomially solvable for $|K| = 1$;

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- We proposed a strengthened MIP formulation - the SM;
- The SM gives a complete description of the associated polyhedron for the single commodity case;
- The SM produces very tight linear programming bounds for the multi-commodity case;
- The SM can, however, be slow to solve, due to large LPs;

FUTURE DEVELOPMENTS

- Analyze the non-convex case.
- Use decomposition methods to reduce the size of the LPs (Benders' + Branch & Cut);
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Thank you for your attention!