Cables network design optimization for the Fiber To The Home

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(1)

DRCN 2016 Paris

(2)

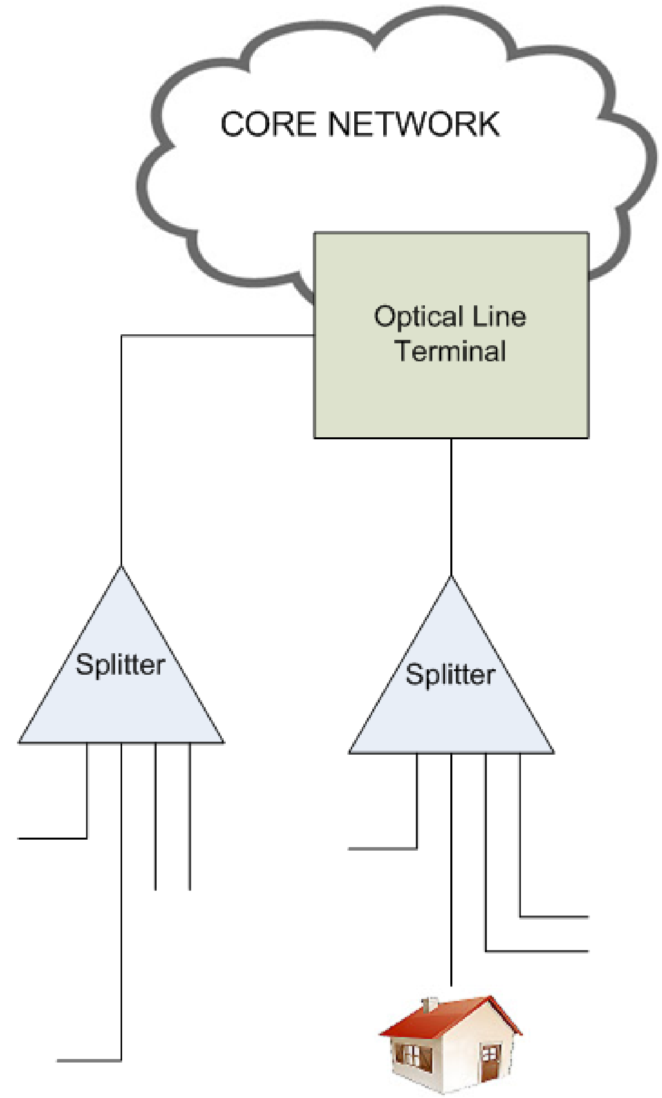
outline of the presentation

outline of the presentation

Fiber To The Home : what is it?

Architecture for more or less dense areas

- last physical part of the fiber network
- connecting households to equipement
- Market conditions make FTTH mandatory
- Technical limit of download speed (« bottleneck ») before fiber optics



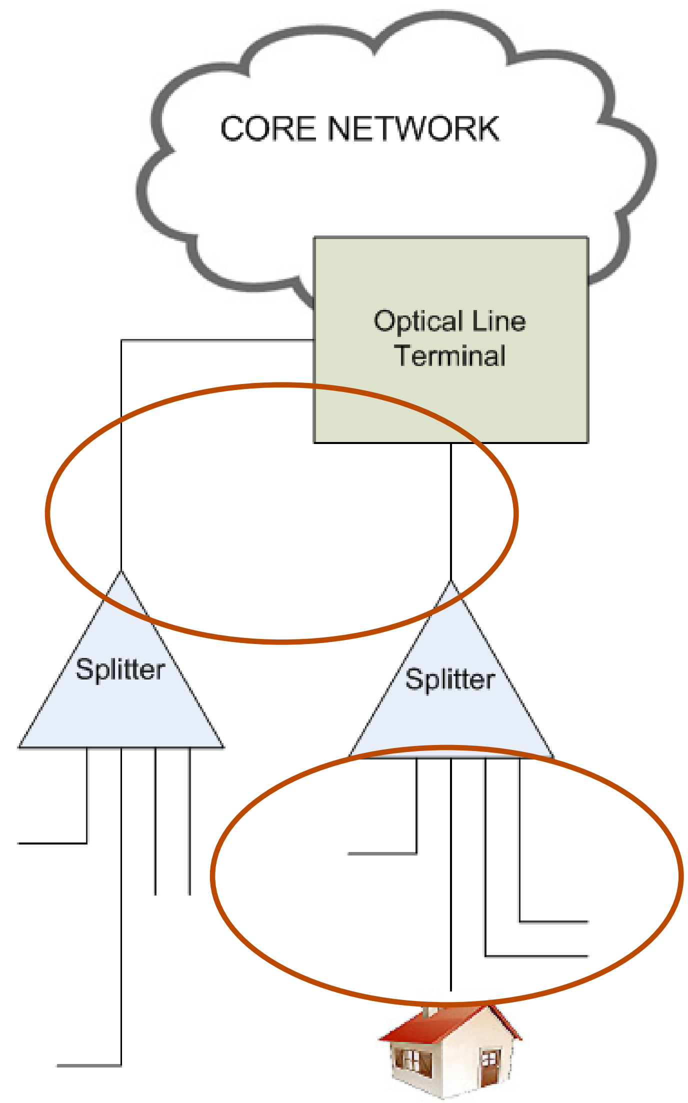
Why care ?

- Huge economical stakes (several billion euros per operator)
- Objective : 100 % of coverage in 2022 in France (cf Mr. Hollande)
- Current coverage : 1.5 million households
- Cables network design appears in different forms for other networks (FTTA, FTTB, FTTC, …)

What has been done so far?

Architecture for more or less dense areas

- Splitter rate
- Splitter location
- Network design, including
  - cable line cost
  - trench digging
- Main limitation is cable modelisation (see survey [1], Axel Werner, Martin Grötschel, Christian Raack)
- Becomes a priority

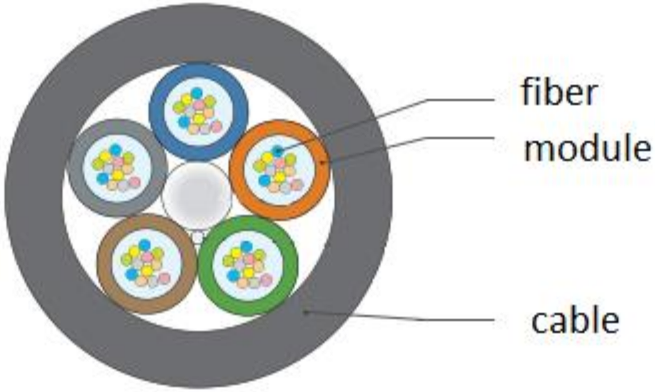


outline of the presentation

Cable anatomy

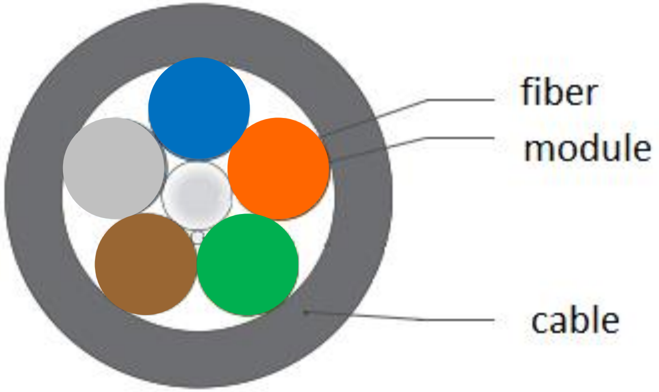
- Three levels :
  - Cable (= several modules)
  - Modules (= several fibers)
  - Fibers (= the goal)
- Not all sizes are available
- Always a fixed number of fiber per modules of the same cable

cross-section of a cable
(5x12 fibers)



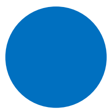
How to consider it?

- Undividable modules (see operations)
- All modules of a given network have the same size (authors context)
- Demands are gathered by modules of demand



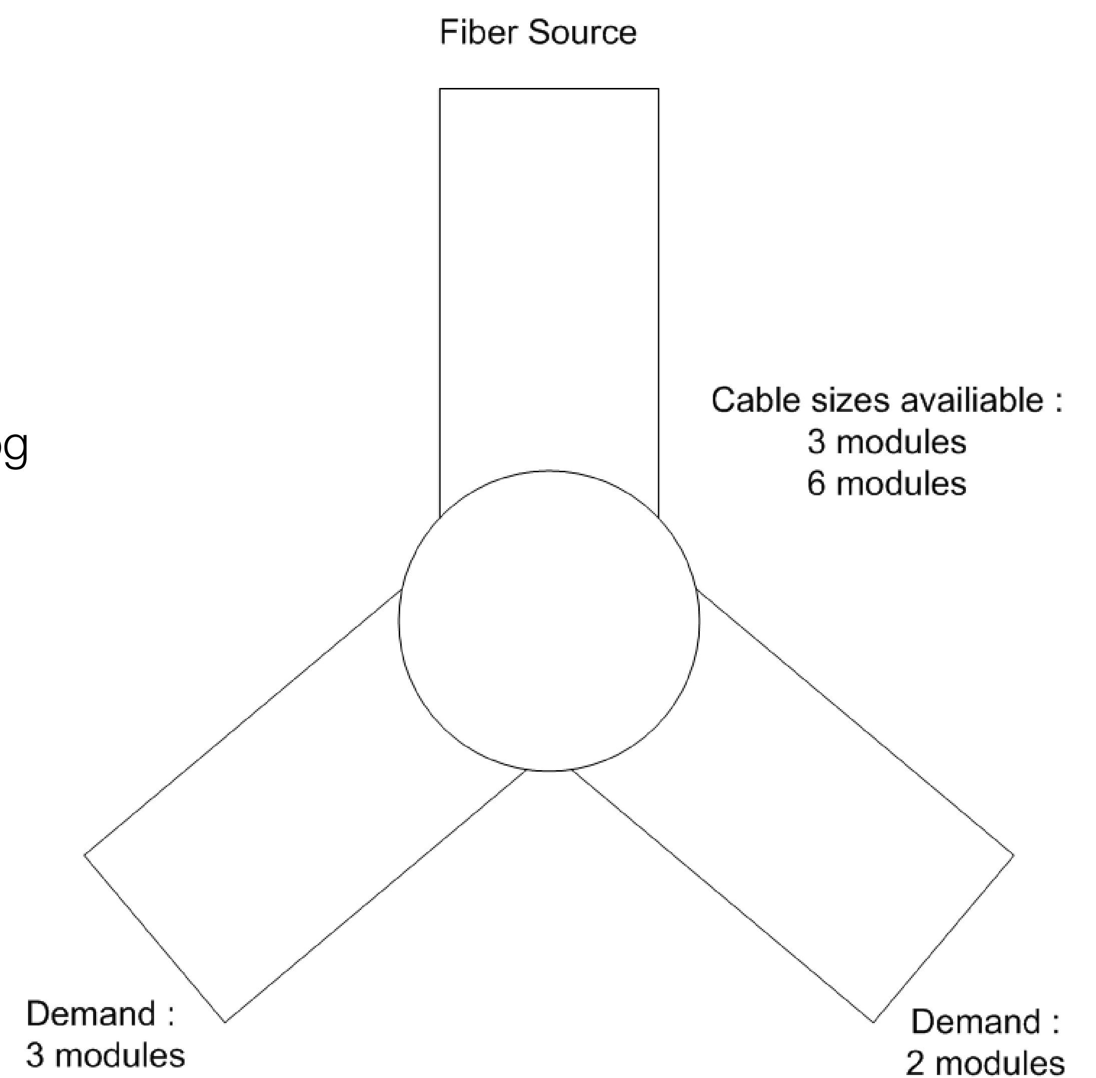
→ we can avoid modeling the fiber level

Demand :



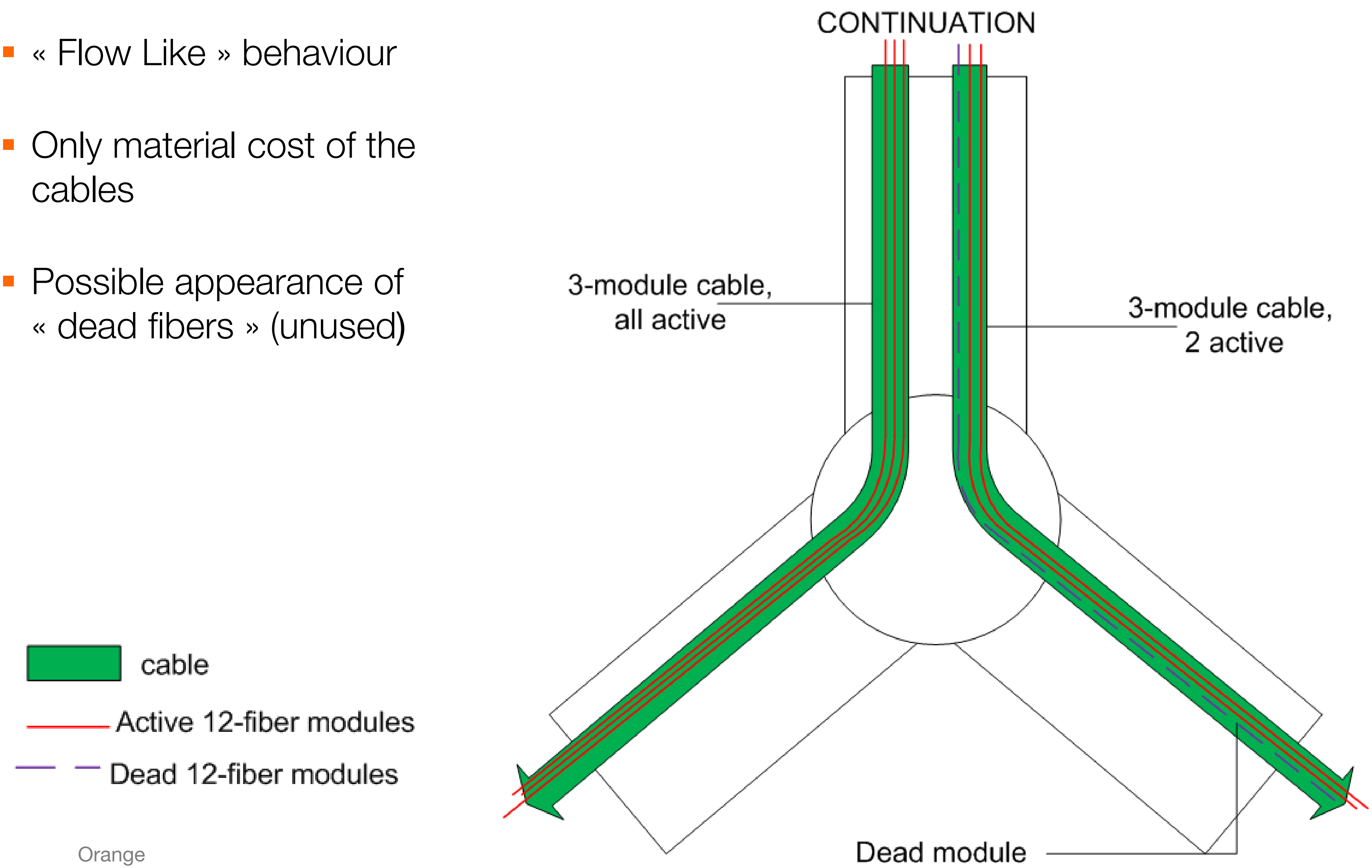
Cable Operational constraints

- Ducts
- Concrete rooms
- demands
- cable manufacturer’s catalog



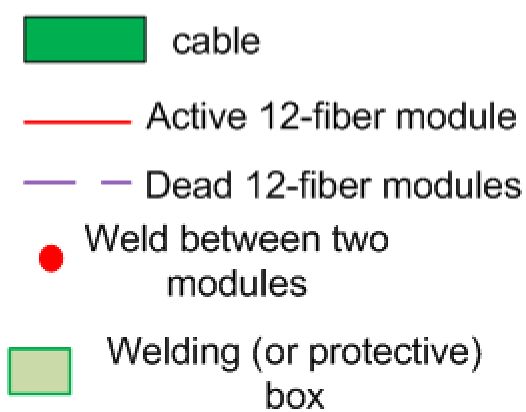
Allowed Operations 1

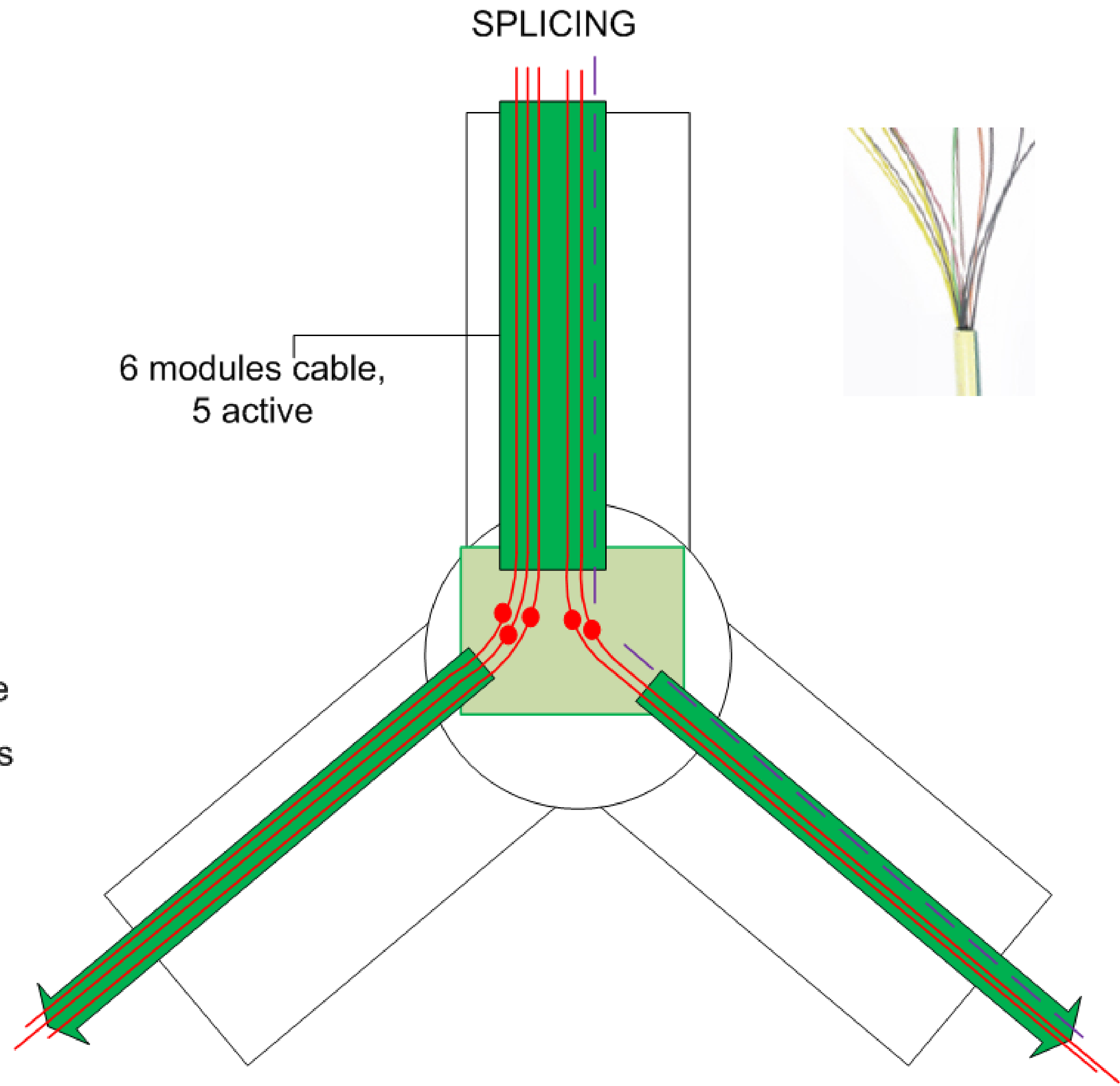
- « Flow Like » behaviour
- Only material cost of the cables
- Possible appearance of « dead fibers » (unused)



Allowed operations 2 : Splicing

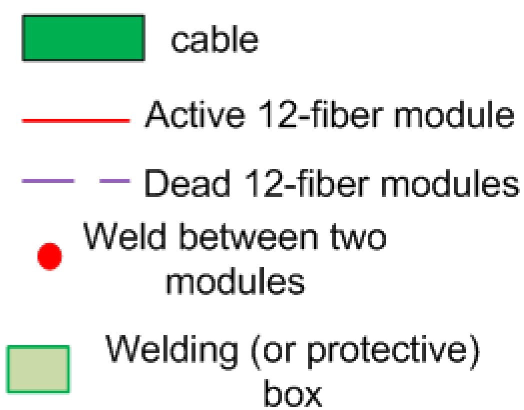
- Material cost of cables
- Material cost of protective box
- Manpower cost of welds (joining fiber modules)
- « Steiner like » behaviour

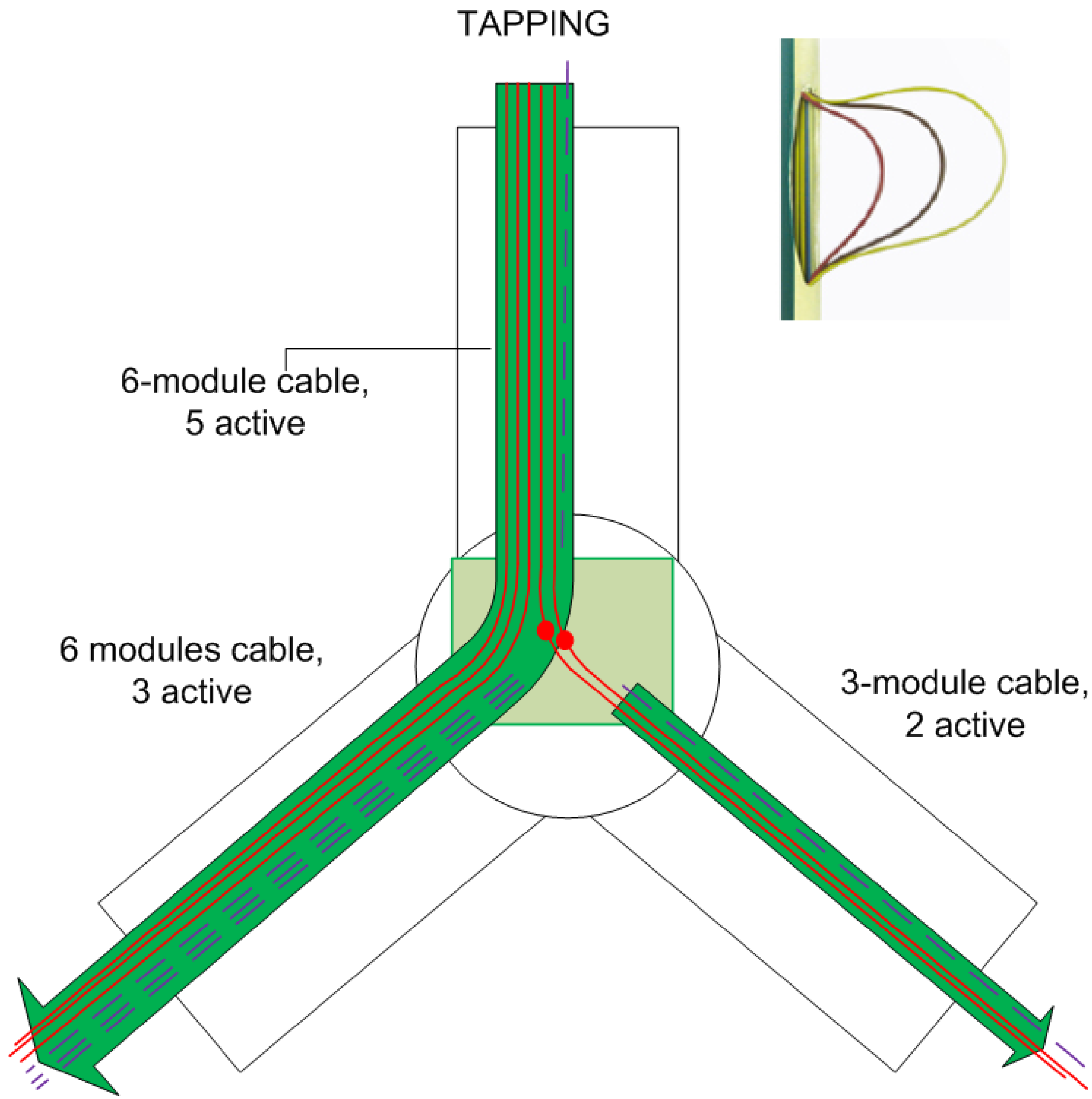




Allowed operations 3 : Tapping

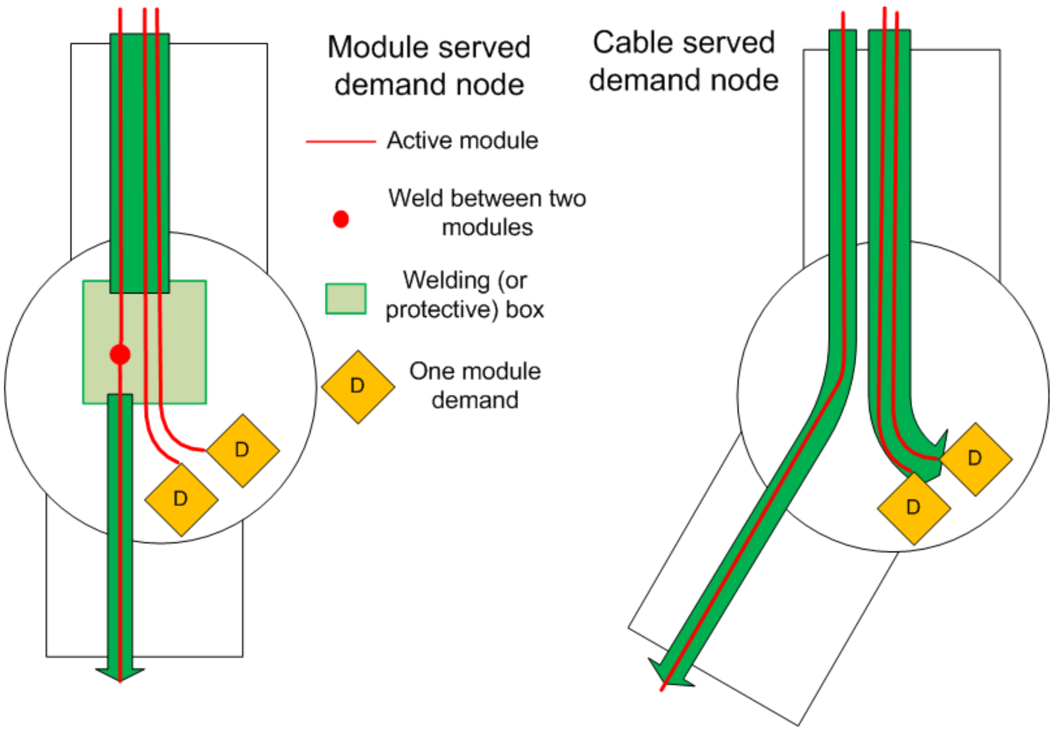
- Material cost of cables
- Material cost of protective box
- Manpower cost of welds





Focus on the demand

- Two ways of satisfying it
- One cable only (economies of scale)
- These two ways cannot be combined, (authors context)
- Additional rule : only one protective box per concrete room (authors context)

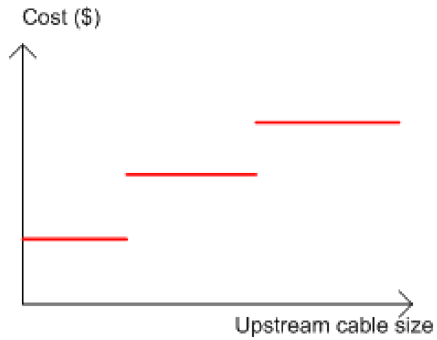


cost shapes

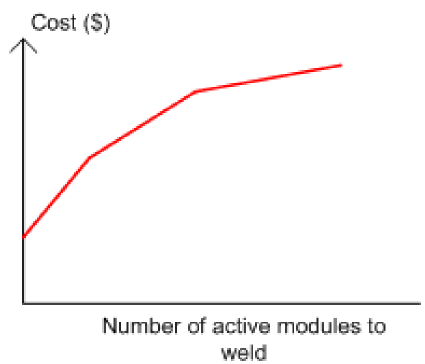
Factured by sub-contractors

- Protective box
  - several sizes
  - piecewise constant
  - material
- Welds
  - concave regarding their number
  - manpower
- Cables
  - concave regarding their diameter
  - material
  - linear regarding their length

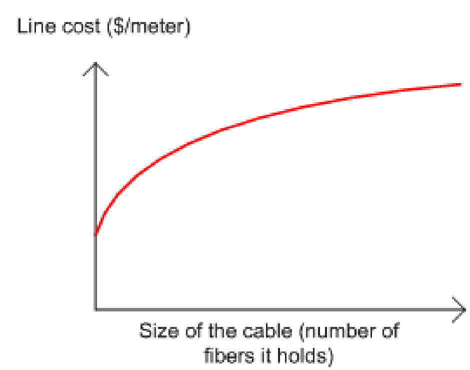
Cost of a splicing box depending on its size



Cost of the welds depending on their number



Line Cost of a cable depending on its size



Problem summary

- Instance
  - ducts and chambers
  - cable list
  - demands at each chamber
  - costs

- Decisions
  - ducts used
  - cables used on each duct
  - number of modules used in each cable
  - action in given node
  - ways of serving the demand

Authors context

- Only one protective box per concrete room (competition regulation)

- Demand served only in one way (marketing)

- Tree topology of the used civil engineering infrastructure (maintenance)

- All fiber modules are identical (maintenance)

outline of the presentation

section 1. Context and motivations

section 2. Problem description

section 3. Integer Programming related elements

section 4. Results and Prospects

Main Inputs

- Graph $G = (V, E)$
- lengths $\{i, j\} \in E \quad d_{i,j}$
- cable set $\mathcal{L} = [1, L]$
- sizes $M_l \text{ with } l \in [1, L]$
- modules $\mathcal{M}_l = [1, M_l]$
- demands $\forall i \in V^*, \; D_i^{mod}$

Variables : arc-node model

- cables : $\forall (i,j,l) \in E \times \mathcal{L}, \forall m \in \mathcal{M}_l, k_{i,j,l,m}$
  - most important variable
  - most « expensive »
- splicing $\forall (i,l) \in V^* \times \mathcal{L}, \forall m \in \mathcal{M}_l, K^{splice}_{i,l,m}$
- tapping $\forall (i,l) \in V^* \times \mathcal{L}, \forall m \in \mathcal{M}_l, K^{tap}_{i,l,m}$
- continuation $\forall (i,l) \in V^* \times \mathcal{L}, \forall m \in \mathcal{M}_l, K^{ctn}_{i,l,m}$
- new cables $\forall (i,l) \in V^* \times \mathcal{L}, \forall m \in \mathcal{M}_l, k^{born}_{i,l,m}$
- number of welds $\forall (i,m) \in V^* \times \mathcal{M}_L, W_{i,m}$
- used edges $\forall (i,j) \in E, X_{i,j}$
- way to serve the demand $\forall i \in V_D, U^{dem}_i$

Model 1

$$
\begin{aligned}
\min \quad & \sum_{(i,j)\in E, l\in\mathcal{L}} \sum_{m\in\mathcal{M}_l} c_l d_{i,j} k_{i,j,l,m} \\
+ \quad & \sum_{i\in V^*, l\in\mathcal{L}} \sum_{m\in\mathcal{M}_l} (K^{spl}_{i,l,m} + K^{tap}_{i,l,m}) PB_l \\
+ \quad & \sum_{i\in V^*, m\in\mathcal{M}_L} W_{i,m} PW_m
\end{aligned}
$$

$$\forall i \in V^*, \sum_{j\in\Gamma(i), l\in\mathcal{L}} \sum_{m\in\mathcal{M}_l} k_{j,i,l,m} m = \sum_{j\in\Gamma(i), l\in\mathcal{L}} \sum_{m\in\mathcal{M}_l} k_{i,j,l,m} m + D^{mod}_i$$

$$\forall (i,l) \in V_N \times \mathcal{L}, \forall m \in \mathcal{M}_l,$$
$$\sum_{j|j\in\Gamma(i)} k_{j,i,l,m} - K^{spl}_{i,l,m} - K^{tap}_{i,l,m} = \sum_{j|j\in\Gamma(i)} k_{i,j,l,m} - k^{born}_{i,l,m} - K^{ctn}_{i,l,m}$$

$$\forall (i,l) \in V_D \times \mathcal{L}, \forall m \in \mathcal{M}_l \setminus \{D^{mod}_i\},$$
$$\sum_{j|j\in\Gamma(i)} k_{i,j,l,m} - k^{born}_{i,l,m} - K^{ctn}_{i,l,m} = \sum_{j|j\in\Gamma(i)} k_{j,i,l,m} - K^{spl}_{i,l,m} - K^{tap}_{i,l,m}$$

$$\forall (i,l) \in V_D \times \mathcal{L}, \forall m \in \{D^{mod}_i\},$$
$$\sum_{j|j\in\Gamma(i)} k_{i,j,l,m} - k^{born}_{i,l,m} - K^{ctn}_{i,l,m} \leq \sum_{j|j\in\Gamma(i)} k_{j,i,l,m} - K^{spl}_{i,l,m} - K^{tap}_{i,l,m}$$

$$\forall i \in V_D, (\sum_{l\in\mathcal{L}} \sum_{m\in\mathcal{M}_l} \sum_{j|j\in\Gamma(i)} k_{i,j,l,m} - k^{born}_{i,l,m} - K^{ctn}_{i,l,m}) + 1 - U^{dem}_i =$$
$$\sum_{l\in\mathcal{L}} \sum_{m\in\mathcal{M}_l} \sum_{j|j\in\Gamma(i)} k_{j,i,l,m} - K^{spl}_{i,l,m} - K^{tap}_{i,l,m}$$

$$\forall (i,l) \in V^* \times \mathcal{L}, \forall m \in \mathcal{M}_l, 0 \leq \sum_{j|j\in\Gamma(i)} k_{i,j,l,m} - k^{born}_{i,l,m} - K^{ctn}_{i,l,m}$$

Model 2

$$\forall i \in V^*, \sum_{l \in \mathcal{L}} \sum_{m \in \mathcal{M}_l} K^{spl}_{i,l,m} + K^{tap}_{i,l,m} \leq \sum_{j \in \Gamma(i)} X_{j,i}$$

$$\forall (i,l) \in V^* \times \mathcal{L}, \sum_{m \in \mathcal{M}_l} K^{ctn}_{i,l,m} = \sum_{m \in \mathcal{M}_l} K^{tap}_{i,l,m}$$

$$\forall i \in V^*, \sum_{l \in \mathcal{L}} \sum_{m \in \mathcal{M}_l} k^{born}_{i,l,m} m = \sum_{m \in \mathcal{M}_L} m W_{i,m}$$

$$\forall i \in V^*, \sum_{m \in \mathcal{M}_L} W_{i,m} \leq 1$$

$$\forall i \in V_N, \sum_{j \in \Gamma(i)} X_{j,i} \leq 1$$

$$\forall i \in V_D, \sum_{j \in \Gamma(i)} X_{j,i} = 1$$

$$\forall (i,j) \in E, X_{i,j} \leq \sum_{l \in \mathcal{L}} \sum_{m \in \mathcal{M}_l} k_{i,j,l,m}$$

$$\forall (i,j) \in E, \sum_{l \in \mathcal{L}} \sum_{m \in \mathcal{M}_l} k_{i,j,l,m} \leq X_{i,j} |V_D|$$

$$X, W, K^{spl}, K^{tap}, K^{ctn}, U^{dem} \in \{0,1\}$$

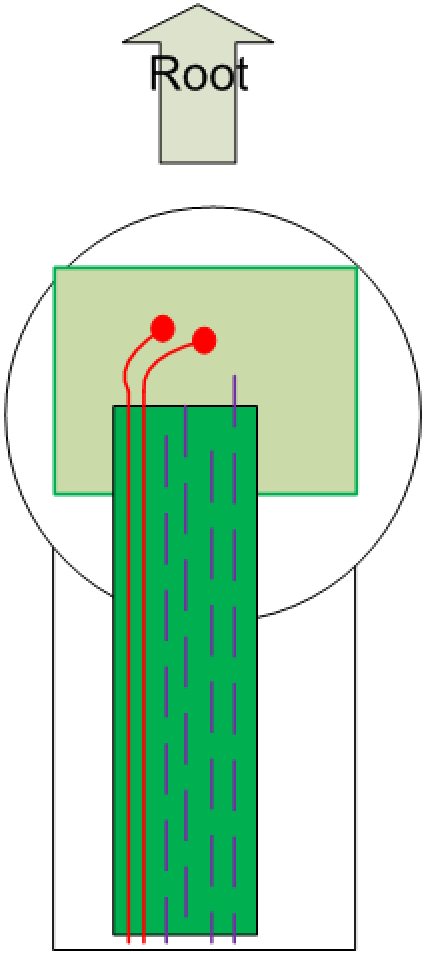
$$k \in [0, |V_D|], k^{born} \in [0, M_L]$$

Variable filtering

- Don't start with a cable bigger than what you really need

$$\forall i \in V^*, \forall l \in \mathcal{L}, \forall m \in \mathcal{M}_{l-1}, k_{i,l,m}^{born} = 0$$

$$\forall (j,l) \in \Gamma(r) \times \mathcal{L}, \forall m \in \mathcal{M}_{l-1}, k_{r,j,l,m} = 0$$



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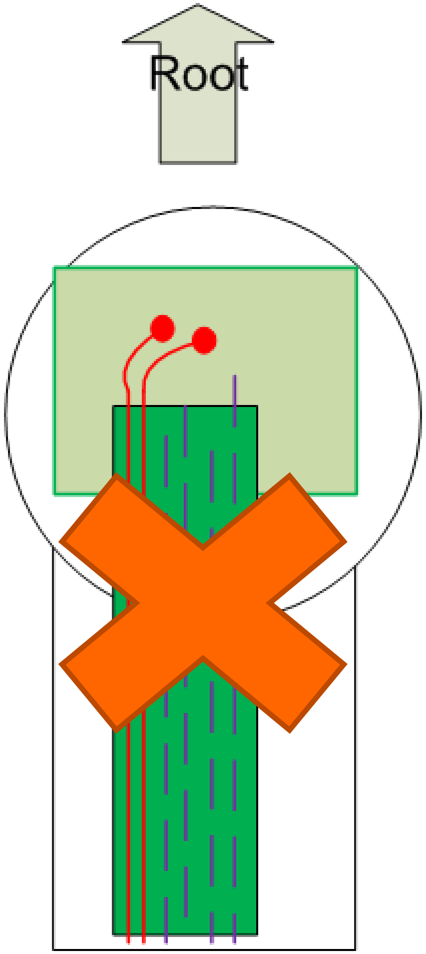

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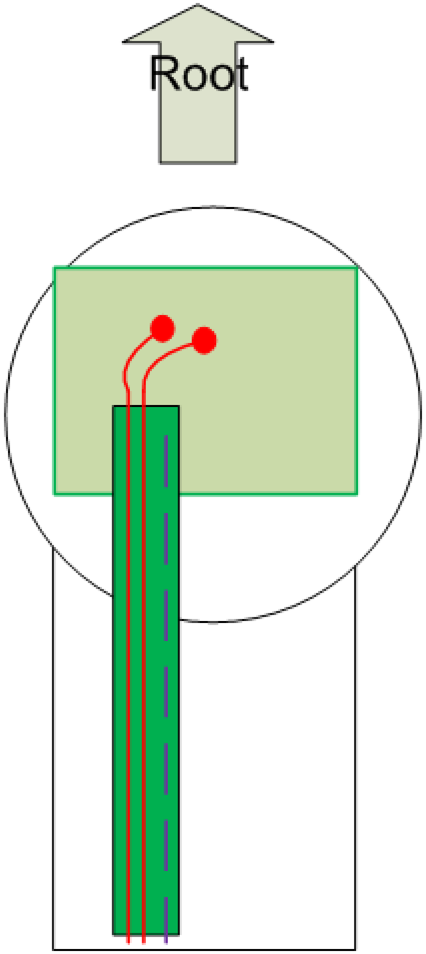

Variable filtering

- Degree one demand-nodes are cable-served
- Only one cable arrives at them

$$\forall i \in V_D,\ \text{if } |\Gamma(i)| = 1,\ \text{then } \forall l \in \mathcal{L}, \forall m \in \mathcal{M}_l,$$
$$k_{i,l,m}^{born} = 0, K_{i,l,m}^{ctn} = 0, K_{i,l,m}^{tap} = 0, K_{i,l,m}^{spl} = 0, u_i^{dem} = 0.$$

Reinforcements

- Gomory-Chvatal cuts

$$\forall m \in \mathcal{M}_L, \sum_{j \in \Gamma(r)} \sum_{l \in \mathcal{L} | m \in \mathcal{M}_l} \sum_{m' \in [m, M_l]} k_{r,j,l,m'} \leq \left\lfloor \frac{\sum_{i \in V_D} D_i^{mod}}{m} \right\rfloor$$

- Steiner Tree related

$$\forall i \in V_N, \quad \sum_{j \in \Gamma(i)} X_{j,i} \leq \sum_{j \in \Gamma(i)} X_{i,j}$$

$$\sum_{(i,j) \in E} X_{i,j} \geq STmin$$

$$\forall (i,j) \in E, \quad X_{i,j} + X_{j,i} \leq 1$$

$$\forall (i,j) \in E, \quad \sum_{j' \in \Gamma(i) \setminus \{j\}} X_{j',i} \geq X_{i,j}$$

- Problem specific valid inequalities

variable number

Initial model (model A)

number of integer variables: $(|V|-1+|E|)\sum_{l\in\mathcal{L}} M_l$

number of boolean variables: $(|V|-1)(3\sum_{l\in\mathcal{L}} M_l + M_L) + |V_D| + |E|$

number of constraints: $(|V|-1)(5+2\sum_{l\in\mathcal{L}} M_l + L) + |V_D|(\sum_{l\in\mathcal{L}} M_l + 1) + 2|E|$

With valid inequalities (model B)

additional constraints:

$$|V_N|(2M_L + \sum_{l\in\mathcal{L}} M_l + 2) + 1 + 2|E| + 2|M_L| + (|V|-1)(2\sum_{l\in\mathcal{L}} M_l + M_L)$$

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Tests on real life instances

- Instance sizes
  - up to 398 edges
  - up to 68 demand nodes
  - up to 78 active modules
  - low degree
  - cable sizes: 1, 2, 4, 6, 8, 12, 18, or 24 modules
- Test set up

Solver : Cplex 12.6.0.0 (Branch and Bound algorithm)

Machine : 4 processors of CPU 5110, 1.6 Ghz each

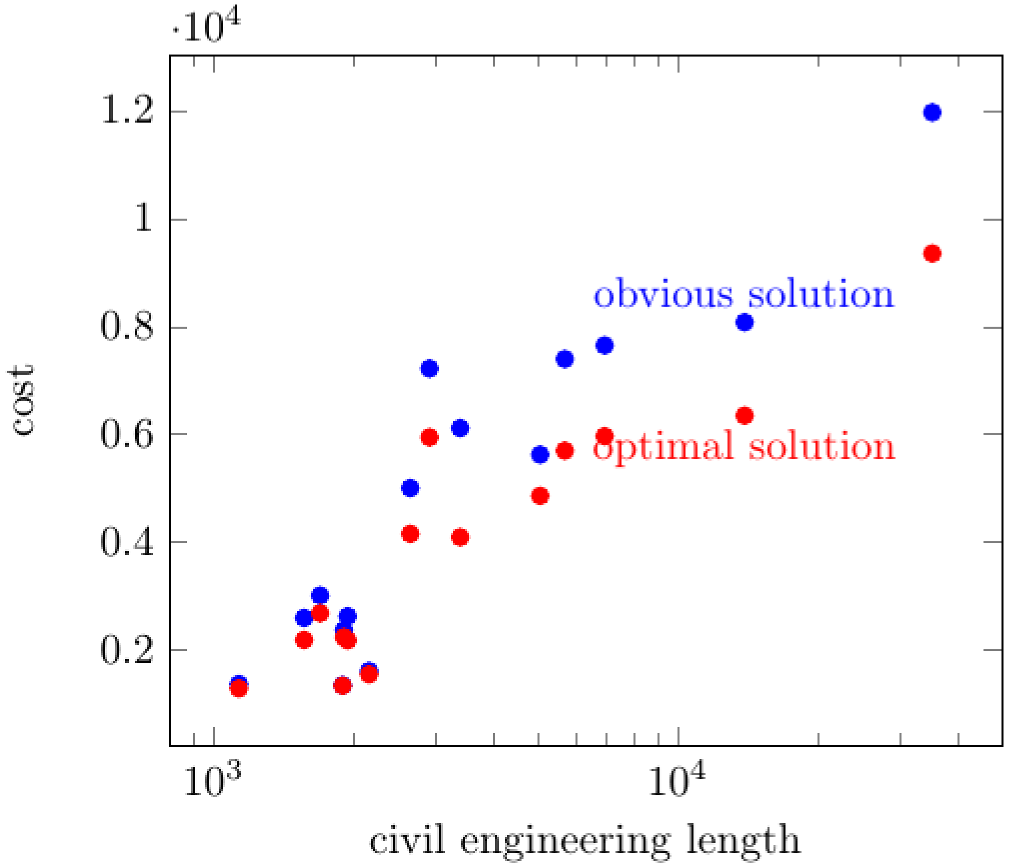
KEY FEATURES OF THE DIFFERENT INSTANCES

instance	nodes	edges	demand nodes	total demand	overall length (m)
zone 0 Cl	83	82	26	38	1566.3
zone 1 Cl	82	85	25	40	1908.2
zone 2 Cl	77	79	24	40	1695.5
zone 3 Cl	70	73	20	28	2161.5
zone 4 Cl	81	87	24	34	1943.4
zone 5 Cl	74	75	22	58	2652.9
zone 6 Cl	57	59	14	20	1132.9
zone 7 Cl	64	64	13	59	1896.0
zone 8 Cl	84	86	21	35	3398.3
zone 0 Ar	127	127	45	61	5697.1
zone 1 Ar	190	220	38	55	35 289.2
zone 2 Ar	128	136	35	66	6941.6
zone 3 Ar	125	124	43	80	2917.1
zone 4 Ar	139	139	44	68	5039.1
zone 5 Ar	168	186	43	67	13 906.5
zone 6 Ar	229	249	35	68	14 525.0
zone 7 Ar	243	270	41	63	35 131.8
zone 8 Ar	353	398	68	78	56 776.8

No optimal solution found for three instances

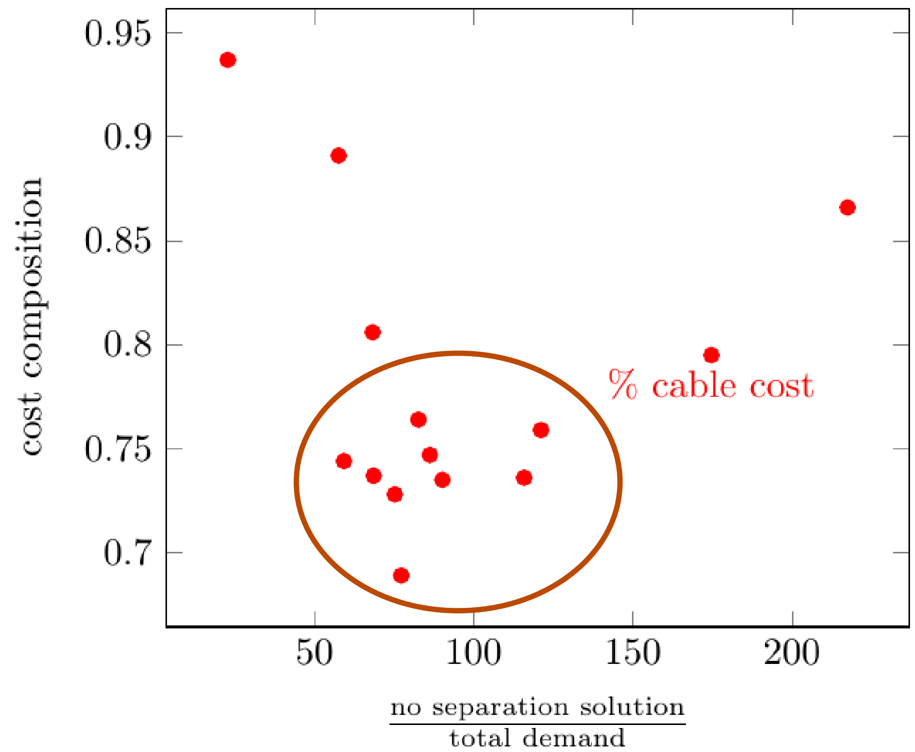
what do we have to win?

- Obvious solution : no splicing, no tapping, all demand nodes are cable-served (shortest paths)
- In average, 20 % savings
- More savings with long distances



Cost composition

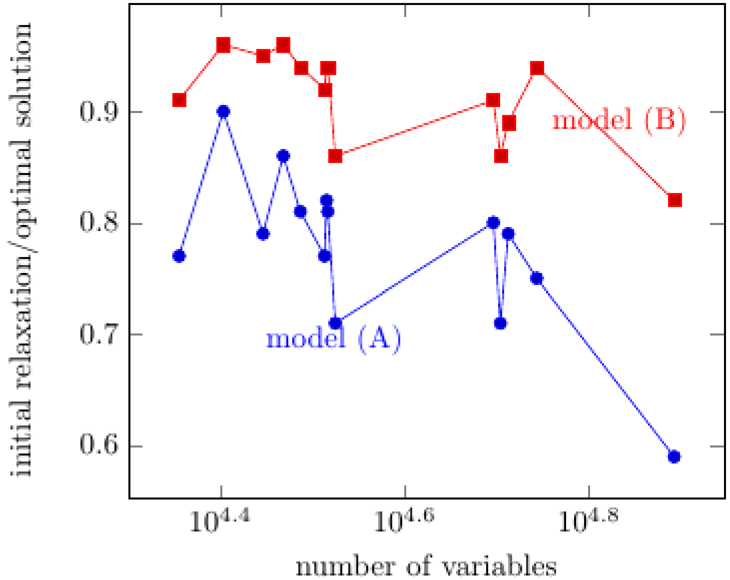
- Average of 20 % of welds and boxes
- Good criteria : solution without splicing or tapping per node
- short distances: flow behaviour
- long distances: steiner tree behaviour

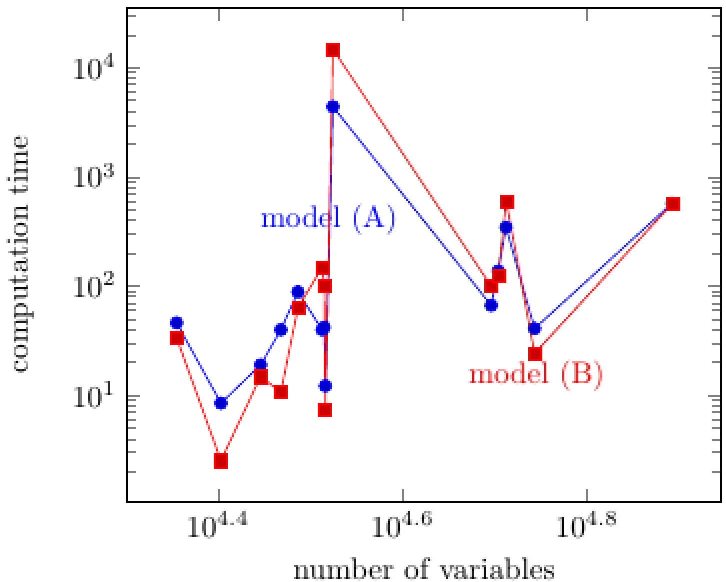


middle length, important separation costs

Influence of valid inequalities

- Good effects on the relaxation
- good relaxation implies good solving time
- Relatively good improvement on performances (better in 60 % of instances, 75 % for first integer solution)

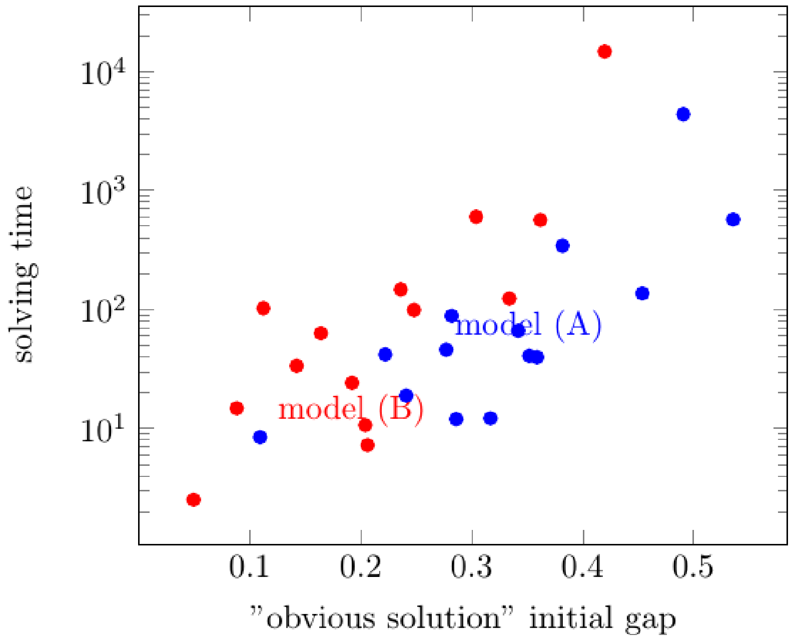




Anticipate trouble

- Initial gap : (solution without splicing or tapping – relaxation)/solution without splicing or tapping

- Good indication of the computation time



Cable set influence

Original set sizes available : 1, 2, 4, 6, 8, 12, 18 or 24 modules
New set sizes available : 1, 2, 6, 12 or 24 modules

- Reduced number of variables (about 40 % less)
- Reduced computation time (about 3 times faster)
- Slightly more expensive (+ 2 %)
- Slightly more material waste (+ 3 %)

Prospects

- Design of heuristic solutions in order to solve the problem on large instances
- Explain odd cases, estimate hard computations
- Be more general (locally imposed rules)
- Different cost functions

thank you